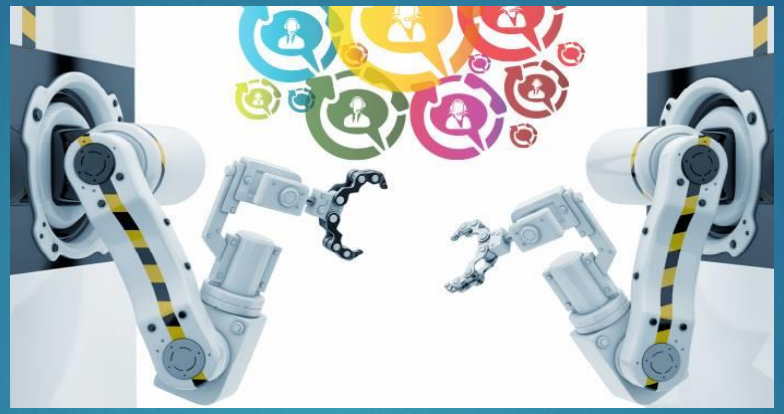
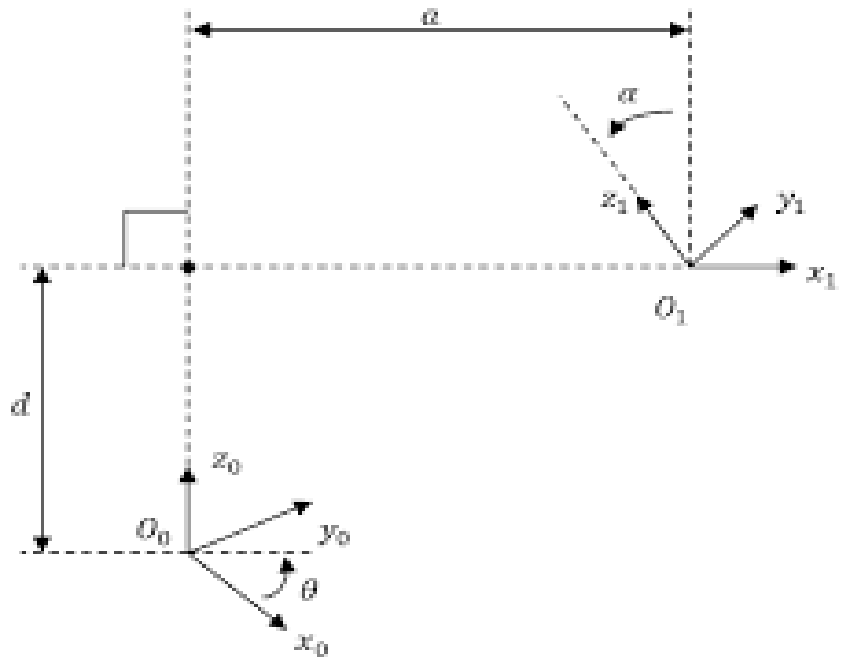


DH-convention Examples

MUHAMMAD A. SHEHAB

Review: DH assumptions

- DH coordinate frame assumptions
  - DH1- The axis x_i is perpendicular to the axis z_{i-1}
  - DH2- The axis x_i intersects the axis z_{i-1}
- Under the above assumptions A_i is achieved by
  1. $Rot(z, \theta_i)$
  2. $Trans(z, d_i)$
  3. $Trans(x, a_i)$
  4. $Rot(x, \alpha_i)$
- $A_i = Rot(z, \theta_i) * Trans(z, d_i) * Trans(x, a_i) * Rot(x, \alpha_i)$



Review: DH parameters

- The parameter a is the distance between the axes z_0 and z_1 measured along the axis x_1
- The parameter α is the angle between the axes z_0 and z_1, measured in a plane normal to x_1 axis
- The parameter d is the perpendicular distance from the origin O_0 to the intersection of the x_1 axis with z_0 axis measured along the z_0 axis
- The parameter θ is the angle between x_0 axis and x_1 axis measured in a plane normal to z_0 axis
- Hint:
  - d: is, only, the variable in case of prismatic joints
  - θ: is, only, the variable in case of revolute joints

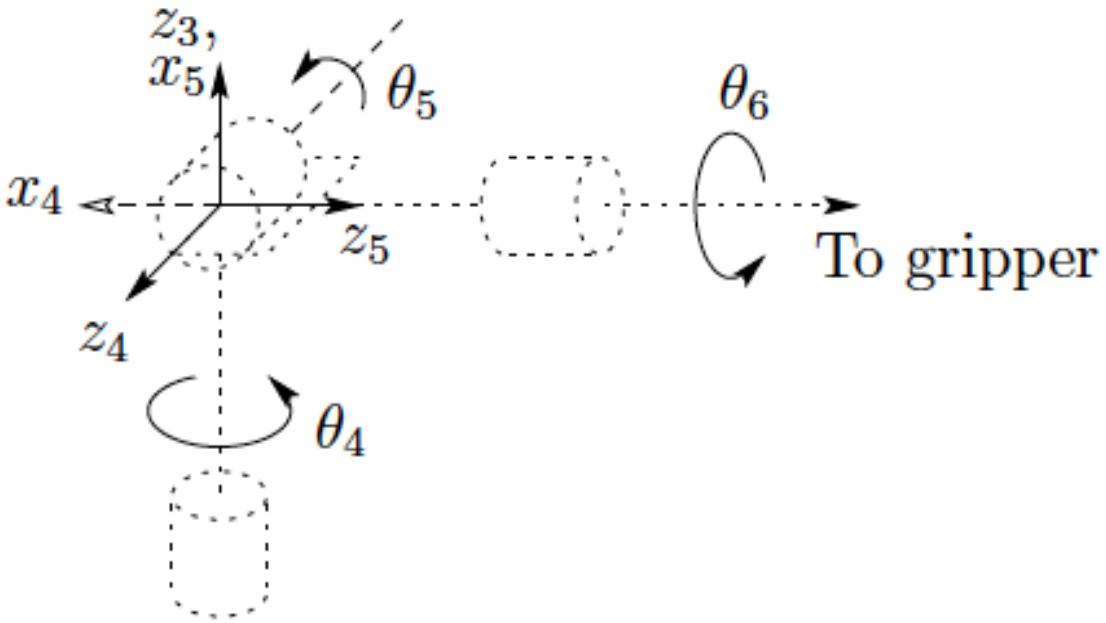
Review: Assignment steps

1. Assign z_i to be axis of actuation of joint $i + 1$
2. Establish arbitrarily the base frame: $x_0, y_0,$ (z_0 determined in step 1)
3. Define x_i based on one of three cases
   a. The axes $z_{i-1}\ and\ z_i$ intersects
   b. The axes $z_{i-1}\ and\ z_i$ are parallel
   c. The axes $z_{i-1}\ and\ z_i$ are not coplanar
4. Define y_i in the appropriate direction to complete the frame
5. The final coordinate frame $o_n x_n y_n z_n$ is commonly referred to as the end-effector or tool frame. The origin O_n is most often placed symmetrically between the fingers of the gripper

Examples

- SPHERICAL WRIST
- CYLINDRICAL MANIPULATOR
- STANFORD MANIPULATOR
- SCARA MANIPULATOR

Spherical Wrist



- The DH Table of Spherical wrist

Link	a_i	α_i	d_i	θ_i
4	0	−90	0	θ_4^*
5	0	90	0	θ_5^*
6	0	0	d_6	θ_6^*

* variable

Spherical Wrist

$$A_4 = \begin{bmatrix} c_4 & 0 & -s_4 & 0 \\ s_4 & 0 & c_4 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_5 = \begin{bmatrix} c_5 & 0 & s_5 & 0 \\ s_5 & 0 & -c_5 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_6 = \begin{bmatrix} c_6 & -s_6 & 0 & 0 \\ s_6 & c_6 & 0 & 0 \\ 0 & 0 & 1 & d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_6^3 = A_4 A_5 A_6$$

$$= \begin{bmatrix} R_6^3 & o_6^3 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} c_4c_5c_6 - s_4s_6 & -c_4c_5s_6 - s_4c_6 & c_4s_5 & c_4s_5d_6 \\ s_4c_5c_6 + c_4s_6 & -s_4c_5s_6 + c_4c_6 & s_4s_5 & s_4s_5d_6 \\ -s_5c_6 & s_5s_6 & c_5 & c_5d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Spherical Wrist vs. Euler Angles

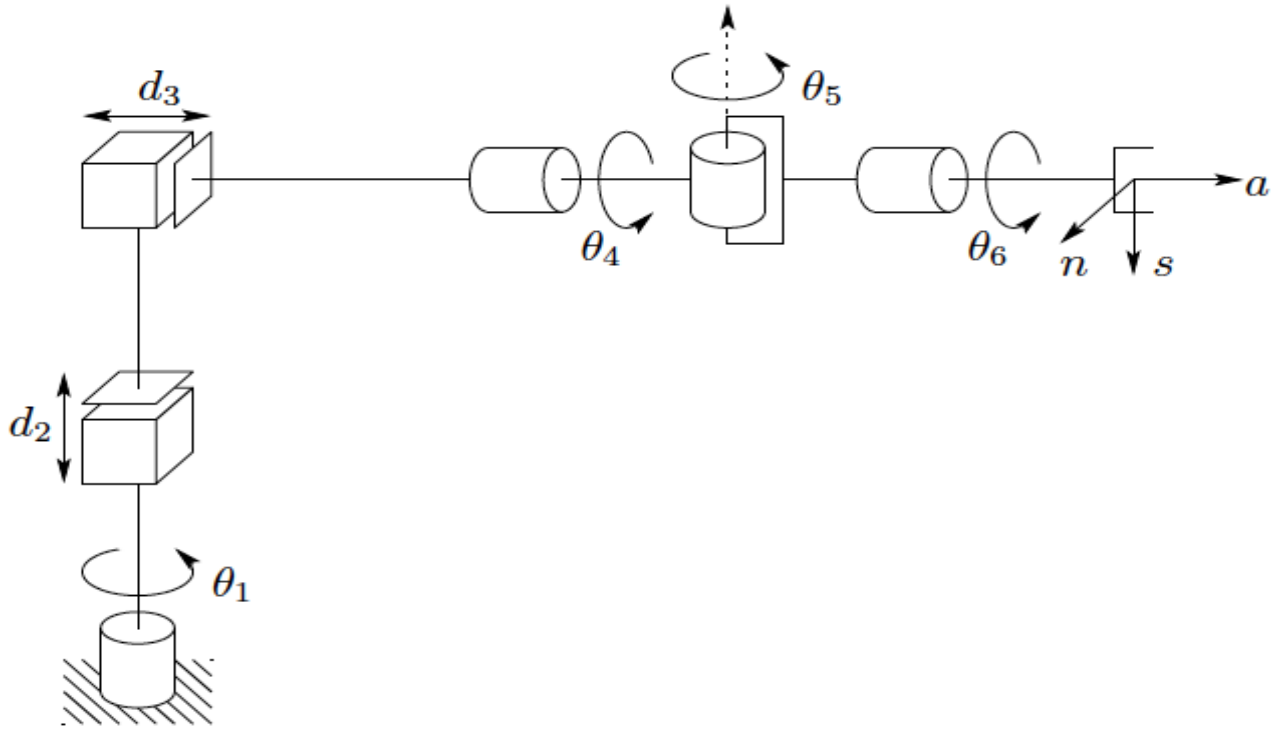
- Spherical wrist is a three-link mechanism for which joint axes z_3, z_4, z_5 intersect at point o called wrist center.
- The final three joint variables $\theta_3, \theta_4, \theta_5$ are the Euler angles $\varphi, \theta, and\ \psi$ respectively, with respect to the coordinate frame $o_3 x_3 y_3 z_3$
- Recall that

$$R_{ZY\,Z} = \begin{bmatrix} c_\phi c_\theta c_\psi - s_\phi s_\psi & -c_\phi c_\theta s_\psi - s_\phi c_\psi & c_\phi s_\theta \\ s_\phi c_\theta c_\psi + c_\phi s_\psi & -s_\phi c_\theta s_\psi + c_\phi c_\psi & s_\phi s_\theta \\ -s_\theta c_\psi & s_\theta s_\psi & c_\theta \end{bmatrix}$$

Cylindrical Manipulator with Spherical Wrist

- Suppose that spherical wrist is attached to cylindrical manipulator in example 2
- The forward kinematics of the whole manipulator

$$T_6^0 = T_3^0 T_6^3$$



Cylindrical Manipulator with Spherical Wrist

$$T_6^0 = \begin{bmatrix} r_{11} & r_{12} & r_{13} & d_x \\ r_{21} & r_{22} & r_{23} & d_y \\ r_{31} & r_{32} & r_{33} & d_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{aligned}
r_{11} &= c_1c_4c_5c_6 - c_1s_4s_6 + s_1s_5c_6 \\
r_{21} &= s_1c_4c_5c_6 - s_1s_4s_6 - c_1s_5c_6 \\
r_{31} &= -s_4c_5c_6 - c_4s_6 \\
r_{12} &= -c_1c_4c_5s_6 - c_1s_4c_6 - s_1s_5c_6 \\
r_{22} &= -s_1c_4c_5s_6 - s_1s_4s_6 + c_1s_5c_6 \\
r_{32} &= s_4c_5c_6 - c_4c_6 \\
r_{13} &= c_1c_4s_5 - s_1c_5 \\
r_{23} &= s_1c_4s_5 + c_1c_5 \\
r_{33} &= -s_4s_5 \\
d_x &= c_1c_4s_5d_6 - s_1c_5d_6 - s_1d_3 \\
d_y &= s_1c_4s_5d_6 + c_1c_5d_6 + c_1d_3 \\
d_z &= -s_4s_5d_6 + d_1 + d_2
\end{aligned}$$

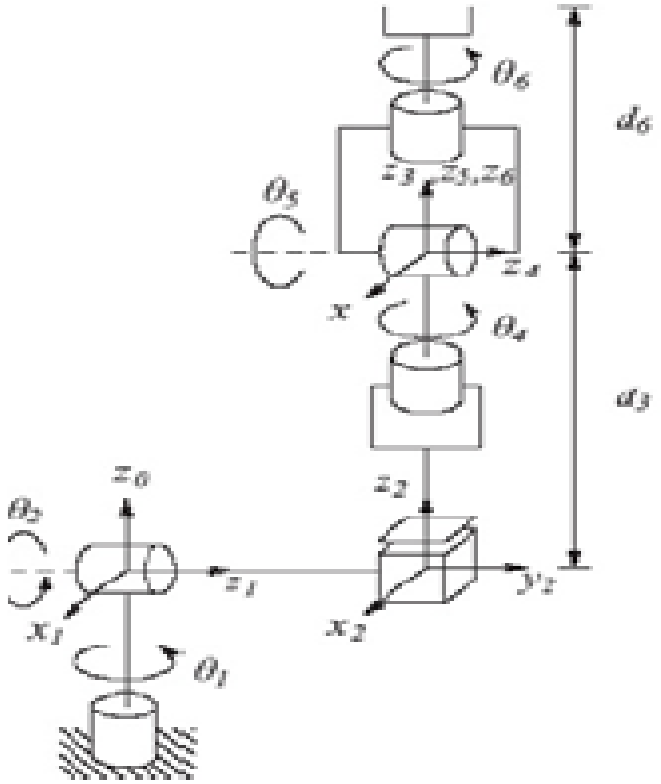
Stanford Manipulator

- This manipulator is an spherical (RRP) manipulator with spherical wrist.

DH parameters for Stanford Manipulator

Link	d_i	a_i	α_i	θ_i
1	0	0	-90	$\theta^\star$
2	d_2	0	$+90$	$\theta^\star$
3	$d^\star$	0	0	0
4	0	0	-90	$\theta^\star$
5	0	0	$+90$	$\theta^\star$
6	d_6	0	0	$\theta^\star$

* joint variable



Stanford Manipulator

$$A_1 = \begin{bmatrix} c_1 & 0 & -s_1 & 0 \\ s_1 & 0 & c_1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} c_2 & 0 & s_2 & 0 \\ s_2 & 0 & -c_2 & 0 \\ 0 & 1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_4 = \begin{bmatrix} c_4 & 0 & -s_4 & 0 \\ s_4 & 0 & c_4 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_5 = \begin{bmatrix} c_5 & 0 & s_5 & 0 \\ s_5 & 0 & -c_5 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_6 = \begin{bmatrix} c_6 & -s_6 & 0 & 0 \\ s_6 & c_6 & 0 & 0 \\ 0 & 0 & 1 & d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Stanford Manipulator

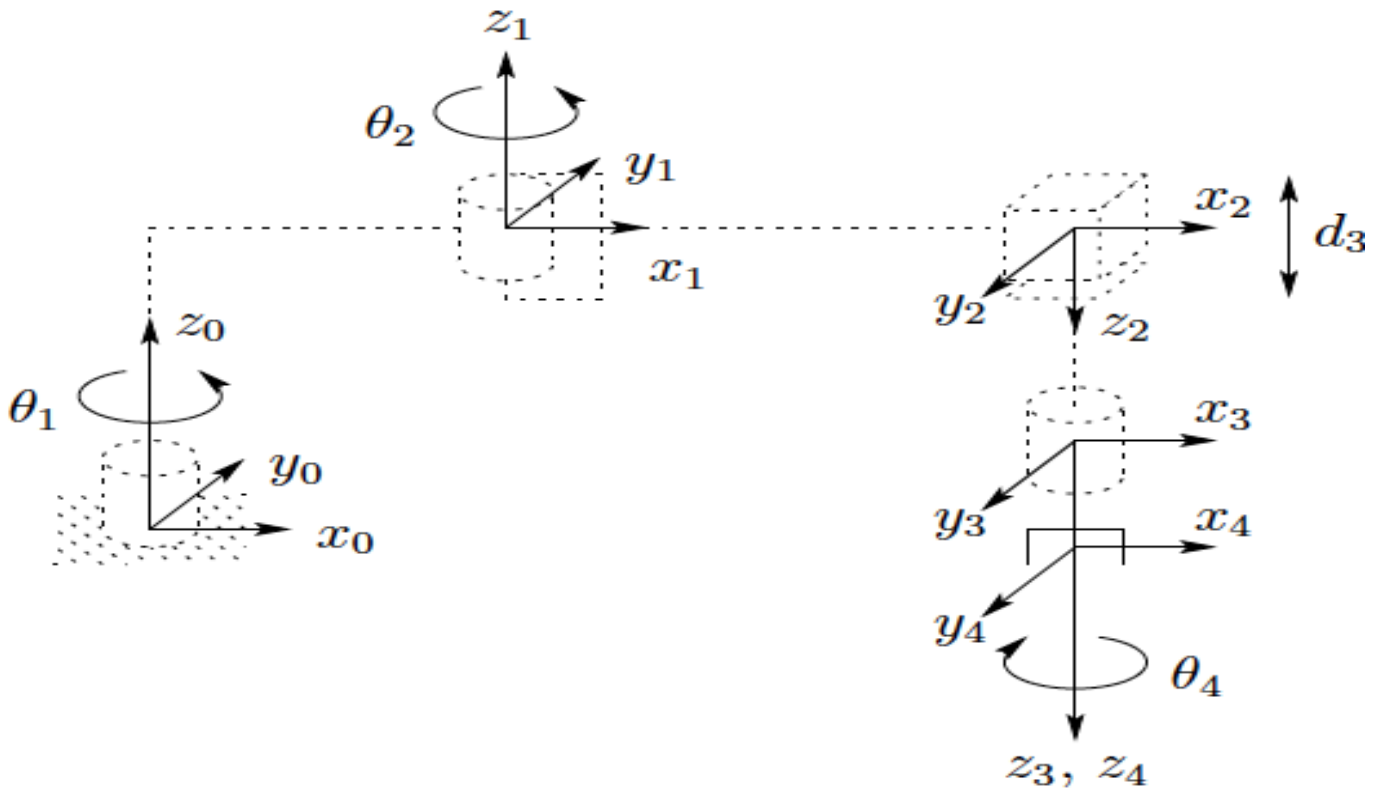
$$
\begin{aligned}
T_6^0 &= A_1 \cdots A_6 \\
&= \begin{bmatrix} r_{11} & r_{12} & r_{13} & d_x \\ r_{21} & r_{22} & r_{23} & d_y \\ r_{31} & r_{32} & r_{33} & d_z \\ 0 & 0 & 0 & 1 \end{bmatrix}
\end{aligned}
$$

$$
\begin{aligned}
r_{11} &= c_1[c_2(c_4c_5c_6 - s_4s_6) - s_2s_5c_6] - d_2(s_4c_5c_6 + c_4s_6) \\
r_{21} &= s_1[c_2(c_4c_5c_6 - s_4s_6) - s_2s_5c_6] + c_1(s_4c_5c_6 + c_4s_6) \\
r_{31} &= -s_2(c_4c_5c_6 - s_4s_6) - c_2s_5c_6 \\
r_{12} &= c_1[-c_2(c_4c_5s_6 + s_4c_6) + s_2s_5s_6] - s_1(-s_4c_5s_6 + c_4c_6) \\
r_{22} &= -s_1[-c_2(c_4c_5s_6 + s_4c_6) + s_2s_5s_6] + c_1(-s_4c_5s_6 + c_4c_6) \\
r_{32} &= s_2(c_4c_5s_6 + s_4c_6) + c_2s_5s_6 \\
r_{13} &= c_1(c_2c_4s_5 + s_2c_5) - s_1s_4s_5 \\
r_{23} &= s_1(c_2c_4s_5 + s_2c_5) + c_1s_4s_5 \\
r_{33} &= -s_2c_4s_5 + c_2c_5 \\
d_x &= c_1s_2d_3 - s_1d_2 + +d_6(c_1c_2c_4s_5 + c_1c_5s_2 - s_1s_4s_5) \\
d_y &= s_1s_2d_3 + c_1d_2 + d_6(c_1s_4s_5 + c_2c_4s_1s_5 + c_5s_1s_2) \\
d_z &= c_2d_3 + d_6(c_2c_5 - c_4s_2s_5)
\end{aligned}
$$

SCARA Manipulator

- This manipulator consists of an RRP arm and a one DOF wrist, whose motion is a roll about the vertical axis

- Report:
  Find the forward kinematics of this manipulator T_4^0



Reference

- Mark W. Spong, Seth Hutchinson and M. Vidyasagar, "Robot Modelling and Control", Wiley, 2005